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A Sliding Mode Observer Design for the Average State Estimation in Large-Scale Systems

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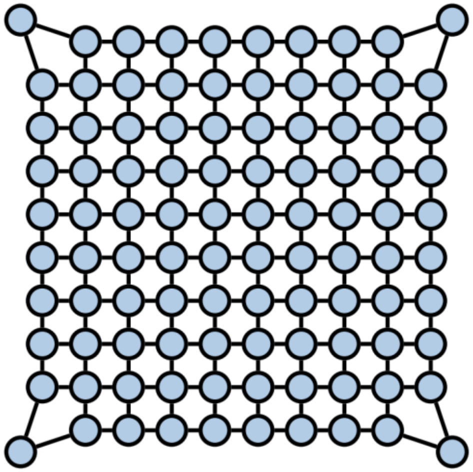
State estimation on LSSs



Consider a Large Scale System (LSS)

$$\Sigma : \begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \\ \boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) \end{cases} \qquad \boldsymbol{x} \in \mathbb{R}^n \ , \ \boldsymbol{y} \in \mathbb{R}^p$$

where $p \ll n$ (much fewer outputs than states)

Full State Estimation Issues on LSS:

- Sometimes is impossible due to the limited number of measurements
- If possible, it may be costly, requiring a dense deployment computational resourses and many sensing gateway devices

To reduce the cost's infrastructure while maintaining tractable the estimators' complexity ***clustered-oriented***, ***decentralized***, and ***functional observers*** are developed

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[*Antoulas 2005,* SIAM book] *Approximation of Large-Scale Dynamical Systems*

[*Pilloni et al. 2013*, IFAC Proc.] *Decentralized state estimation in connected systems*

[*Niazi et al. 2020,* IEEE TCNS] *Average state estimation in large-scale clustered network systems*

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Motivations for the average state estimation

- In many LSS monitoring applications, it is often sufficient/preferred the estimation of some ***aggregated functionals*** of the **system states**, such as their ***mean***

$$a(t) = \frac{1}{k}\mathbf{1}_k^\top \boldsymbol{x}_u(t) \quad \text{with} \quad \boldsymbol{x}_u \in \mathbb{R}^k \subset \mathbb{R}^n,\ k = n - p \quad (\textit{unavailable states})$$

- This is particularly of interest, if the **LSS is not observable** and consists a coupled subsystems with commensurable states (e.g. reaction-diffusion, or compartmental, or multi-agents systems)

List of application of interest:

- Estimation of the *mean traffic volume* in portions of a road map/communication network

  [*Coogan et al.*2015, IEEE TAC] *A compartmental model for traffic networks and its dynamical behaviour*

- Estimation of the *mean circulation of leaders opinions* over a social network

  [*Li et al.2017*, IOP Series of Earth Env. Sci.] *Modelling opinion transmission on social networks under opinion leaders*

- Estimation of the *mean temperature* in smart buildings through a network of sensors

  [*Deng et al.2010*, Proc. of IEEE ACC] *Building thermal model reduction via aggregation of states*

- Estimation of the *mean proportion of infected people* during the spreading of an epidemic

  [*Martin et al.2020*, IEEE TNSE] *Subgraph detection for average detectability of LTI systems.*

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Average state estimation problem (1)

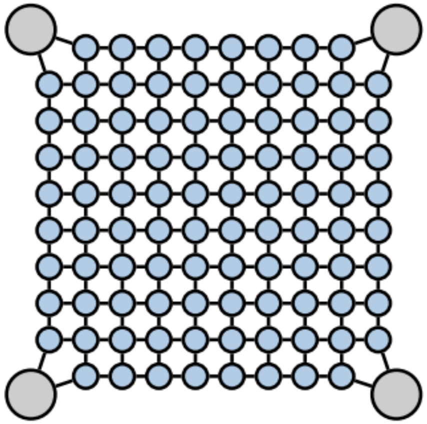
- Consider an LSS consisting of n coupled dynamic subsystems, with commensurable states

$$\dot{x}_i(t) = \sum_{j=1}^{n} a_{ij} x_j(t) + \sum_{\ell=1}^{m} b_{i\ell} u_\ell(t)$$

(scalar only for simplicity in the notation)

where only $p \ll n$ nodes are equipped with sensing gateway devices

- The LSS can thus be partitioned into measured ◯ and unmeasured ○ nodes



$$\begin{bmatrix}\dot{\boldsymbol{x}}_u \\ \dot{\boldsymbol{y}}\end{bmatrix} = \overbrace{\begin{bmatrix}\boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22}\end{bmatrix}}^{\boldsymbol{A}\in\mathbb{R}^{n\times n}} \begin{bmatrix}\boldsymbol{x}_u \\ \boldsymbol{y}\end{bmatrix} + \overbrace{\begin{bmatrix}\boldsymbol{B}_1 \\ \boldsymbol{B}_2\end{bmatrix}}^{\boldsymbol{B}\in\mathbb{R}^{n\times m}} \boldsymbol{u}$$

$$\boldsymbol{y} = \underbrace{\begin{bmatrix}\boldsymbol{0}_{p\times k} & \boldsymbol{I}_p\end{bmatrix}}_{\boldsymbol{C}\in\mathbb{R}^{p\times n}} \begin{bmatrix}\boldsymbol{x}_u \\ \boldsymbol{y}\end{bmatrix}$$

$$\boldsymbol{x} \in \mathbb{R}^n \mapsto \begin{bmatrix}\boldsymbol{x}_u \\ \boldsymbol{y}\end{bmatrix}$$

$$\boldsymbol{x}_u \in \mathbb{R}^k,\ \boldsymbol{y} \in \mathbb{R}^p$$

$$p \ll k = n - p$$

Problem formulation

- Let the **LSS** be **neither *observable* nor *detectable*** (*to avoid trivial cases*), our aim is to design an observer that, by means of $\boldsymbol{y}$ and $\boldsymbol{u}$ allows the estimation of the average of $\boldsymbol{x}_u(t)$.

$$a(t) = \frac{1}{k}\,\boldsymbol{1}_k^\top\,\boldsymbol{x}_u(t)$$

(mean of $\boldsymbol{x}_u(t)$)

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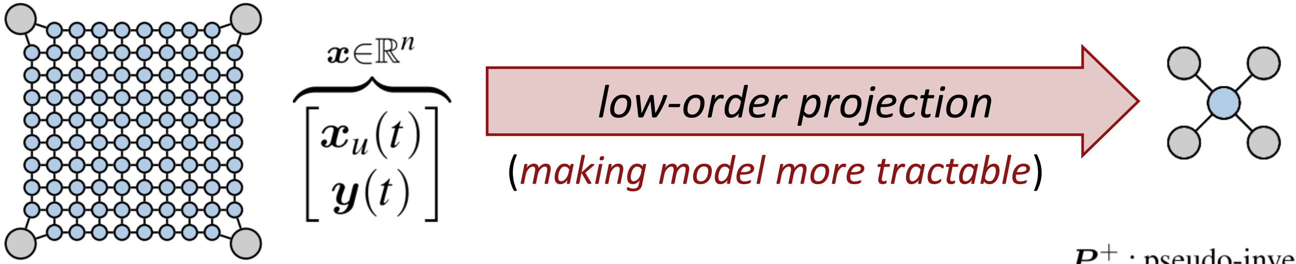
Average state estimation problem (2)

Existing approaches exploits ***low-order projections*** and ***functional observers*** to design ***linear open-loop estimators*** [*Niazi 2019*, IEEE L-CSS] and ***Luenberger-like observers*** [*Sadamoto 2017*, IEEE TCNS]



$$\overbrace{\begin{bmatrix}\boldsymbol{x}_u(t)\\ \boldsymbol{y}(t)\end{bmatrix}}^{\boldsymbol{x}\in\mathbb{R}^n}$$

$$\overbrace{\begin{bmatrix}a(t)\\ \boldsymbol{y}(t)\end{bmatrix}}^{\boldsymbol{z}\in\mathbb{R}^{p+1}} = \overbrace{\begin{bmatrix}\frac{1}{k}\mathbf{1}_k^\top & \mathbf{0}_p^\top\\ \mathbf{0}_{p\times k} & \boldsymbol{I}_p\end{bmatrix}}^{\boldsymbol{P}\in\mathbb{R}^{(p+1)\times n}} \overbrace{\begin{bmatrix}\boldsymbol{x}_u(t)\\ \boldsymbol{y}(t)\end{bmatrix}}^{\boldsymbol{x}\in\mathbb{R}^n}$$

Since $\boldsymbol{P}$ is rectangular the ***inverse-projection*** $\begin{bmatrix}\boldsymbol{x}_u(t)\\ \boldsymbol{y}(t)\end{bmatrix} = \overbrace{\boldsymbol{P}^\top\left(\boldsymbol{P}\boldsymbol{P}^\top\right)^{-1}}^{\boldsymbol{P}^+\,:\,\text{pseudo-inverse}} \begin{bmatrix}a(t)\\ \boldsymbol{y}(t)\end{bmatrix}$ **losses information!!!!**

Such missed information **will bother us** in term of the ***state-dependent Unknown Input*** (UI) "$\boldsymbol{F}\boldsymbol{\delta}(t)$"

$$\begin{bmatrix}\dot{a}(t)\\ \dot{\boldsymbol{y}}(t)\end{bmatrix} = \underbrace{\begin{bmatrix}\frac{1}{k}\mathbf{1}_k^\top\boldsymbol{A}_{11}\mathbf{1}_k & \frac{1}{k}\mathbf{1}_k^\top\boldsymbol{A}_{12}\\ \boldsymbol{A}_{21}\mathbf{1}_k & \boldsymbol{A}_{22}\end{bmatrix}}_{\boldsymbol{E}=\boldsymbol{P}\boldsymbol{A}\boldsymbol{P}^+\in\mathbb{R}^{(p+1)\times(p+1)}} \begin{bmatrix}a(t)\\ \boldsymbol{y}(t)\end{bmatrix} + \underbrace{\begin{bmatrix}\frac{1}{k}\mathbf{1}_k^\top\boldsymbol{B}_1\\ \boldsymbol{B}_2\end{bmatrix}}_{\boldsymbol{G}=\boldsymbol{P}\boldsymbol{B}\in\mathbb{R}^{(p+1)\times m}} \boldsymbol{u}(t) + \begin{bmatrix}\boldsymbol{F}_1\\ \boldsymbol{F}_2\end{bmatrix} \underbrace{\left(\boldsymbol{I}_k - \frac{1}{k}\mathbf{1}_k\cdot\mathbf{1}_k^\top\right)\boldsymbol{x}_u(t)}_{\substack{\boldsymbol{\delta}(t)\in\mathbb{R}^{p+1}\,:\,\mathbf{1}_k^\top\boldsymbol{\delta}(t)=0\\ \text{(unknown input)}}}$$

The presence of ***matched & unmatched UIs*** makes us think

Sliding-Mode Observers (**SMOs**) and ***output injection concepts***

could be used to design a ***more performant nonlinear observer***

$\boldsymbol{\delta}$ is the deviation of $\boldsymbol{x}_u(t)$ from $a(t)$

$$\mathbf{1}^\top\boldsymbol{\delta}(t) = 0$$

[*Niazi et al. 2019*, IEEE L-CSS] *Scale-free estimation of the average state in large-scale systems*

[*Sadamoto et al. 2017*, IEEE TCNS] *Average state observers for large-scale network systems*

Proposed Average SMO design

- Reduced-order LSS model

(unmatched UI term)

$$\begin{bmatrix}\dot{a}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}E_{11} & E_{12}\\ E_{21} & E_{22}\end{bmatrix}\begin{bmatrix}a\\ y\end{bmatrix} + \begin{bmatrix}G_1\\ G_2\end{bmatrix}u + \begin{bmatrix}F_1\delta\\ F_2\delta\end{bmatrix} \qquad (1)$$

(matched UI term)

- Proposed ***Average Sliding-Mode Observer***

$$\begin{bmatrix}\dot{\hat{a}}\\ \dot{\hat{y}}\end{bmatrix} = \begin{bmatrix}E_{11} & E_{12}\\ E_{21} & E_{22}\end{bmatrix}\begin{bmatrix}\hat{a}\\ \hat{y}\end{bmatrix} + \begin{bmatrix}G_1\\ G_2\end{bmatrix}u + \overbrace{\begin{bmatrix}-E_{12}\\ E_{22}^{s} - E_{22}\end{bmatrix}}^{H}\overbrace{(\hat{y} - y)}^{e_y} + \begin{bmatrix}L\\ -I_p\end{bmatrix}\overbrace{\rho\,\mathrm{sign}\,(y - \hat{y})}^{\nu}$$

Differently to ***standard SMO formulations***, our design **account** also ***unmatched UIs*** due to $F_1\delta(t)$

Assumption 1 (The LSS is average observable): The pair (E, CP^{+}) is observable.

Assumption 2 (The LSS has bounded states): One of the following holds:

$$\mathrm{eig}(A) \subset \mathbb{C}_{\leq 0} \text{ and } \int_0^\infty \|u(t)\| dt < \infty \qquad \text{OR} \qquad \mathrm{eig}(A) \subset \mathbb{C}_{<0} \text{ and } \|u(t)\| < \infty \; \forall \; t \geq 0$$

Theorem 1 *(Exact average state estimation)*

Consider the ***LSS's lower order projection*** (1) and let ***Assumptions 1-2*** be in force.

If and only if the following conditions holds $\mathrm{rank}\left(\begin{bmatrix}\alpha 1_k^\top - F_1\\ F_2\end{bmatrix}\right) = \mathrm{rank}\,(F_2)$

The following **tuning**

$$E_{22}^{s} \in \mathbb{C}_{<0}^{p} \qquad \rho > \|E_{21}a\|_\infty + \|F_2\delta\|_\infty \qquad L = \left(\alpha\, 1_k^\top - F_1\right) F_2^{+} \qquad \alpha < \frac{(F_1(F_2^{+}F_2 - I_k)1_k)}{(1_k^\top F_2^{+} F_2 1_k)} \subseteq \mathbb{R}$$

guarantees the **exact asymptotic estimation** of "$a(t)$"

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Proof sketch of Theorem 1 (exact estimation)

- Let $\begin{cases} \boldsymbol{e}_a = \boldsymbol{a} - \hat{\boldsymbol{a}} & \textit{(average estimation error)} \\ \boldsymbol{e}_y = \boldsymbol{y} - \hat{\boldsymbol{y}} & \textit{(output estimation error)} \end{cases}$ then $\begin{cases} \dot{\boldsymbol{e}}_a = E_{11}\boldsymbol{e}_a - \boldsymbol{F}_1\boldsymbol{\delta} + \boldsymbol{L}\boldsymbol{\nu} \\ \dot{\boldsymbol{e}}_y = \boldsymbol{E}_{21}\boldsymbol{e}_a + \boldsymbol{E}^s_{22}\boldsymbol{e}_y - \boldsymbol{F}_2\boldsymbol{\delta} - \boldsymbol{\nu} \end{cases}$, $\quad \boldsymbol{E}^s_{22} \in \mathbb{C}^p_{<0}$, $\boldsymbol{\nu} = \rho\,\mathrm{sign}(\boldsymbol{e}_y)$

Thanks to the ***given SMO design*** the ***average estimation error*** is **decoupled from outputs**

- Due to the ***Assumption 2*** (i.e. ***LSS stability***) the stabilization of "$\boldsymbol{e}_y(t)$" can be studied separately

IF → $\rho > \|\boldsymbol{E}_{21}\boldsymbol{a}\|_\infty + \|\boldsymbol{F}_2\boldsymbol{\delta}\|_\infty$ → *Finite-time stability of* $\boldsymbol{e}_y(t)$ → $\frac{d}{dt}\overbrace{\|\boldsymbol{e}_y(t)\|_2^2}^{V(t)} < -\varepsilon\,\overbrace{\|\boldsymbol{e}_y(t)\|_2}^{\sqrt{V(t)}}$

$\overset{\boldsymbol{0}_p}{\cancel{\dot{\boldsymbol{e}}_y}} = \boldsymbol{E}_{21}\boldsymbol{e}_a + \boldsymbol{E}^s_{22}\overset{\boldsymbol{0}_p}{\cancel{\boldsymbol{e}_y}} - \boldsymbol{F}_2\boldsymbol{\delta} - \boldsymbol{\nu}_{eq}$ *THEN* → $\boldsymbol{\nu}_{eq} = \overbrace{\boldsymbol{E}_{21}\boldsymbol{e}_a - \boldsymbol{F}_2\boldsymbol{\delta}}^{\text{equivalent information}}$ ⇒ $\dot{\boldsymbol{e}}_a = \overbrace{(E_{11} + \boldsymbol{L}\boldsymbol{E}_{21})}^{\text{Is it negative?}}\boldsymbol{e}_a - \overbrace{(\boldsymbol{F}_1 + \boldsymbol{L}\boldsymbol{F}_2)\boldsymbol{\delta}}^{\text{Can it be zero?}}$

- Because of $\boldsymbol{\delta}(t) : \boldsymbol{1}_k^\top\,\boldsymbol{\delta}(t) \equiv 0$ → *Rouché-Capelli theorem* → $\mathrm{rank}\left(\begin{bmatrix} \alpha\boldsymbol{1}_k^\top - \boldsymbol{F}_1 \\ \boldsymbol{F}_2 \end{bmatrix}\right) = \mathrm{rank}(\boldsymbol{F}_2)$

$\exists\, \boldsymbol{L} = (\alpha\,\boldsymbol{1}_k^\top - \boldsymbol{F}_1)\,\boldsymbol{F}_2^+ \;:\; \boxed{\boldsymbol{F}_1 + \boldsymbol{L}\boldsymbol{F}_2 = \alpha\boldsymbol{1}_k^\top}\;\forall\,\alpha \in \mathbb{R}$ ⇒ $\dot{\boldsymbol{e}}_a = \overbrace{(E_{11} + \boldsymbol{L}\boldsymbol{E}_{21})}^{\text{Is it negative?}}\boldsymbol{e}_a - \overbrace{\alpha\boldsymbol{1}_k^\top\boldsymbol{\delta}}^{=0}$

- Consider the resulting internal dynamic by simple manipulations

$$\dot{\boldsymbol{e}}_a = \Big(\overbrace{E_{11}}^{\boldsymbol{F}_1\boldsymbol{1}_k} + \underbrace{\boldsymbol{L}}_{(\alpha\,\boldsymbol{1}_k^\top - \boldsymbol{F}_1)\boldsymbol{F}_2^+}\overbrace{E_{21}}^{\boldsymbol{F}_2\boldsymbol{1}_k}\Big)\boldsymbol{e}_a = \beta(\alpha)\boldsymbol{e}_a$$

IF → $\alpha < \bar{\alpha} = \dfrac{(\boldsymbol{F}_1(\boldsymbol{F}_2^+\boldsymbol{F}_2 - \boldsymbol{I}_k)\boldsymbol{1}_k)}{(\boldsymbol{1}_k^\top\boldsymbol{F}_2^+\boldsymbol{F}_2\boldsymbol{1}_k)} \subseteq \mathbb{R}$

THEN → $\beta(\alpha) < 0$

■

Extension to generic LSSs (1)

- **Theorem 1** states that $\lim_{t\to\infty} e_a(t) = 0$ IF

$$\boxed{\operatorname{rank}\left(\begin{bmatrix}\alpha\mathbf{1}_k^\top - \boldsymbol{F}_1\\ \boldsymbol{F}_2\end{bmatrix}\right) = \operatorname{rank}\left(\boldsymbol{F}_2\right)} \qquad (2)$$

- Although ***LSSs satisfying (2) exists*** [*Niazi et al. 2019*,IEEE L-CSS] it is a **very restrictive requirement**

Theorem 2 *(Relaxed average state estimation)*

Consider the ***LSS's lower order projection***, ***Assumption 1-2*** and the ***Average-SMO*** of ***Theorem 1.***

Suppose the **LSS** does **not satisfy** the **rank constraint** (2).

Then, "$e_a(t)$" remain ***bounded*** and ***converges*** to $\limsup_{t\to\infty}\left|e_a(t)\right| \leq \dfrac{\|\boldsymbol{F}_1 + \boldsymbol{L}(\alpha)\boldsymbol{F}_2 - \alpha\mathbf{1}_k^\top\|}{|\beta(\alpha)|} \times \|\boldsymbol{\delta}(t)\|$

*where the **boundary layer size** can be minimized by* $\alpha^* = \operatorname*{argmin}_{\alpha<\bar{\alpha}} \dfrac{\|\boldsymbol{F}_1 + \boldsymbol{L}(\alpha)\boldsymbol{F}_2 - \alpha\mathbf{1}_k^\top\|}{|\beta(\alpha)|}$

Proof's sketch: From the ***Proof of Theorem 1*** $\xrightarrow{\;e_y = 0\,\forall\, t > T^*\;}$

$$\dot{e}_a = \underbrace{(E_{11} + \boldsymbol{L}\boldsymbol{E}_{21})}_{\beta(\alpha)<0} e_a - \underbrace{(\boldsymbol{F}_1 + \boldsymbol{L}\boldsymbol{F}_2)\boldsymbol{\delta}}_{\substack{\text{No more zero,}\\ \text{but the effect}\\ \text{are minimized!}}}$$

- Because of $\boldsymbol{\delta}(t) \,:\, \mathbf{1}_k^\top\,\boldsymbol{\delta}(t) \equiv 0$ and ***Projection's Theorem*** [*Luenberger 1997*, Wiley]

$$\boldsymbol{L} = \underbrace{\left(\alpha\mathbf{1}_k^\top - \boldsymbol{F}_1\right)\boldsymbol{F}_2^+}_{\text{least-square minimizer}} \equiv \operatorname*{argmin}_{\boldsymbol{L}\in\mathbb{R}^{1\times p}} \|\boldsymbol{F}_1 + \boldsymbol{L}\boldsymbol{F}_2 - \alpha\mathbf{1}_k^\top\| \quad \forall\,\alpha$$

- Finally note that

$$\limsup_{t\to\infty}\left|\mathrm{e}^{\beta(\alpha)t}e_a(0) + \int_0^t \mathrm{e}^{\beta(\alpha)(t-\tau)}(\boldsymbol{F}_1 + \boldsymbol{L}\boldsymbol{F}_2)\boldsymbol{\delta}(t)d\tau\right| \leq \overbrace{\frac{\|\boldsymbol{F}_1 + \boldsymbol{L}(\alpha)\boldsymbol{F}_2 - \alpha\mathbf{1}_k^\top\|}{|\beta(\alpha)|}}^{\alpha=\alpha^*\text{minimizes this term}} \times \|\boldsymbol{\delta}(t)\| \qquad \blacksquare$$

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Estimation precision VS Convergence rate

- The previous discussion states that $\limsup_{t\to\infty}\left|e_a(t)\right| \le \frac{\|\boldsymbol{F}_1+\boldsymbol{L}(\alpha)\boldsymbol{F}_2-\alpha\mathbf{1}_k^\top\|}{|\beta(\alpha)|}\times\|\boldsymbol{\delta}(t)\|$

  is minimized by $\alpha^* = \operatorname*{argmin}_{\alpha<\bar{\alpha}\subset\mathbb{R}} \frac{\|\boldsymbol{F}_1+\boldsymbol{L}(\alpha)\boldsymbol{F}_2-\alpha\mathbf{1}_k^\top\|}{|\beta(\alpha)|}$

- Moreover, by simple manipulation one further derives that

$$\alpha^* = \operatorname*{argmin}_{\alpha<\bar{\alpha}\subset\mathbb{R}} \frac{\|\boldsymbol{F}_1+\boldsymbol{L}(\alpha)\boldsymbol{F}_2-\alpha\mathbf{1}_k^\top\|}{|\beta(\alpha)|} = \cdots\cdots = \operatorname*{argmin}_{\alpha<\bar{\alpha}\subset\mathbb{R}} \frac{\|(\boldsymbol{F}_1-\alpha\mathbf{1}_k^\top)(\boldsymbol{I}_k-\boldsymbol{F}_2^+\boldsymbol{F}_2)\|}{\left|(\boldsymbol{F}_1-\alpha\mathbf{1}_k^\top)(\boldsymbol{I}_k-\boldsymbol{F}_2^+\boldsymbol{F}_2)\mathbf{1}_k^\top+k\alpha\right|}$$

Although α^* minimize the **steady-state average error upper-bound**, its optimal choice

may have adverse **effects** on the **convergence rate** $\beta(\alpha)$ of the **estimation error dynamics**

where

$$\dot{e}_a = \overbrace{(E_{11}+\boldsymbol{L}(\alpha)\boldsymbol{E}_{21})}^{\beta(\alpha)<0} e_a - (\boldsymbol{F}_1+\boldsymbol{L}(\alpha)\boldsymbol{F}_2)\boldsymbol{\delta} \qquad \boldsymbol{L} = \left(\alpha\,\mathbf{1}_k^\top - \boldsymbol{F}_1\right)\boldsymbol{F}_2^+$$

- To **trade-off** those **2 performance metrics**, the next simple but effective adaptation law is proposed

$$\dot{\alpha}(t) = \begin{cases} -\lambda & \text{if } \left|\left|e_y(t)\right|\right| < \varepsilon \text{ and } \bar{\alpha} > \alpha(t) \ge \alpha^* \\ 0 & \text{otherwise} \end{cases} \quad \text{where} \quad \begin{array}{l} \lambda > 0 \\ 0 < \varepsilon < \|e_y(0)\| \end{array}$$

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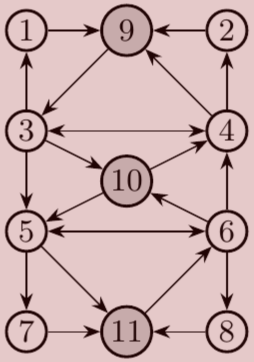
Numerical results

Case 1 A **compartmental system** consisting of [*Walter et al. 2012*,Springer]

$$n = 11, p = 3$$

compartments whose interconnections are as in the given **directed graph**

Rank Cond. [**YES**]

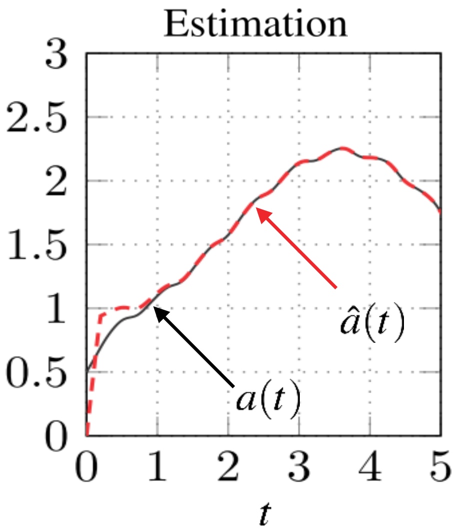


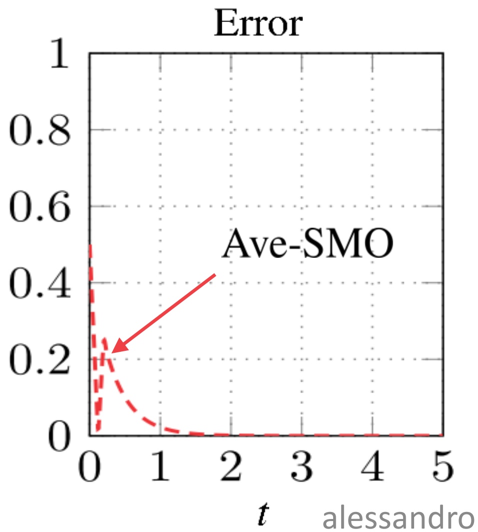
SMO Tuning: $\rho = 5, \boldsymbol{E}_{22}^s = -10\boldsymbol{I}_3, \alpha : \beta(\alpha) \in \mathbb{R}_{<0}$

- Since the rank condition holds

$$\dot{e}_a = \overbrace{(E_{11} + \boldsymbol{L}(\alpha)\boldsymbol{E}_{21})}^{\beta(\alpha)} e_a - \cancelto{\mathbf{0}_k}{(\boldsymbol{F}_1 + \boldsymbol{L}\boldsymbol{F}_2)\boldsymbol{\delta}}$$

- Then $\boldsymbol{L}$ is chosen arbitrarily to perform a desired convergence rate, e.g., $\beta = -3$



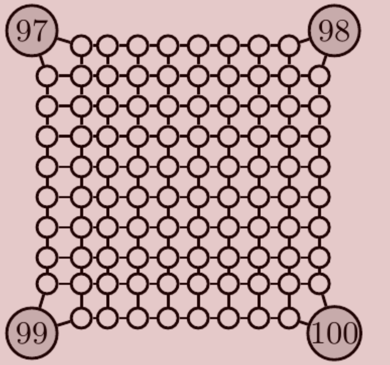


Case 2 A **reaction-diffusion process** consisting of [*Arcak 2011*,Automatica]

$$n = 100, p = 4$$

substances which diffusion is modelled as the given **undirected graph**

Rank Cond. [**NO**]

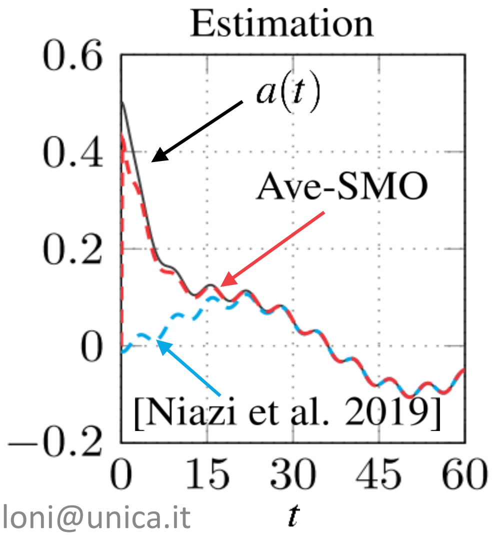


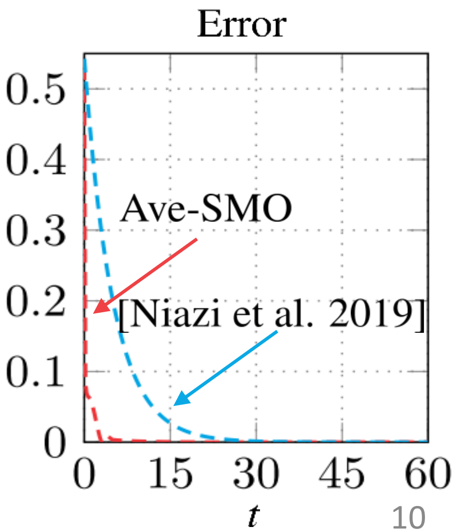
SMO tuning: $\rho = 5, \boldsymbol{E}_{22}^s = -10\boldsymbol{I}_4, \alpha^* = -2.1 \times 10^{-3}$

- Since the rank condition does not holds

$$\dot{e}_a = (E_{11} + \boldsymbol{L}(\alpha)\boldsymbol{E}_{21})e_a - (\boldsymbol{F}_1 + \boldsymbol{L}\boldsymbol{F}_2)\boldsymbol{\delta}$$

- The Average-SMO is now compared with *the **linear observer*** in [*Niazi et al. 2019*,IEEE L-CSS] which exploits also a **different optimal tuning**





Concluding remarks

Main results:

- The **average-state estimation problem** on LSSs with many unmeasurable nodes is **solved** as the design of a ***reduced-order SMO*** subjected to **matched** and **unmatched perturbations**
- **IMPORTANT:** Differently to the existing approaches our **Average-SMO** allows to ***resort information on the UIs*** to **improve** the ***robustness***, ***precision*** and ***convergence rate*** of estimation
- **IMPORTANT:** Our design is general, and **it can be used without** modification on LSSs independently to the fact they satisfy or not the exact estimation condition

$$\operatorname{rank}\left(\begin{bmatrix}\alpha\mathbf{1}_k^{\top}-\boldsymbol{F}_1\\ \boldsymbol{F}_2\end{bmatrix}\right)=\operatorname{rank}\left(\boldsymbol{F}_2\right)$$

- The **observer complexity** is independent to the number of unavailable nodes, thus keeping the design scalable and computationally treatable even if the system size is large.

Hints for future investigations:

- **Decentralized** and **clustered-oriented versions** of the proposed design **are currently under study**
- Alternative **fixed-time estimation strategies** appear also promising and **worthy of further investigation**

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Reference:

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Thank you for your kind attention

Any question, suggestion, etc. is welcome…

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