Coordination of multi-agent systems: stability via nonlinear Perron-Frobenius theory and consensus for desynchronization and dynamic estimation

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This thesis considers analysis and control problems in complex large networks composed by multiple interacting agents, called multi-agent systems (MASs).

One **major result** of the thesis is the development of a general theory for the analysis of a class of nonlinear MASs. The literature of nonlinear MASs is vast and it mostly consists of ad-hoc solutions based on Lyapunov theory, in constrast, the **innovative aspect** of our work is that it provides a novel approach which exploits recent nonlinear extensions of Perron-Frobenius theory. Even though techniques of nonlinear Perron-Frobenius theory have been broadly exploited in several fields, its use in the domain of MASs has not been previously explored as far as we know. This is quite peculiar, given that most of the theory for linear MASs strongly relies on classical Perron-Frobenius theory, thus motivating our research direction and suggesting that it breaks new ground for future theoretical developments. The **theoretical contributions** are synthesized next:

1) The identification of a novel class of nonlinear maps and the convergence analysis of their iterative behavior, namely *type-K order-preserving* and *homogeneous* maps: their forward orbits are proved to eventually converge to a fixed point of the map, if any exists. It is remarkable that nonnegative row-stochastic matrices, pivotal in the consensus analysis of linear MASs, turn out to be a special case.

2) The application of the above technical result to the analysis of MASs both in discrete and continuous time. Necessary and sufficient conditions on the agents' dynamics and their reciprocal interaction are derived in order to guarantee that the interconnected system falls into the above mentioned class, thus guaranteeing its global stability with respect to the set of equilibrium points.

3) The link between the developed theory and graph theory: it is proved that standard connectivity assumptions on the communication network are sufficient to restrict the set of equilibrium points of a MAS to consensus, thus steering the agents toward the same state value.

It is clear that the range of **possible applications** is wide due to the general nature of the approach. Indeed, applications discussed within this thesis are max-consensus protocols, epidemics over networks, bounded control input, synchronization of phase oscillators. Moreover, other applications currently under investigation include opinion dynamics, Krasnolseskii-Mann algorithms, chemical reaction networks.

An additional **major result** is the design of novel distributed algorithms which solve specific problems and are useful for several applications.

In particular we consider the problem of breaking synchronization in networks of diffusively coupled harmonic oscillators. We propose a distributed state feedback which solves the problem in connected networks with arbitrary undirected interactions. The **theoretical contributions** include the definition of a novel desynchronization measure, whose definition is not standard as the thoroughly investigated notion of synchronization, and its linkage to the Fiedler eigenvector of the underling Laplacian matrix. Indeed, the **innovative aspect** of our distributed state feedback is that it makes use if a novel protocol for the distributed estimation of the Fiedler vector of the Laplacian matrix, which constitutes another original result of our work. Among the **possible applications**, a networked mechanical system consisting of a certain number of wagons with linear dumper interaction and a mass-spring-damper modeling the interaction with the rails. It is interesting to notice that in this example, desynchronization assumes a clear physical meaning, in fact it corresponds to the minimum stress on the rails.

Another problem addressed in the thesis is the estimation of the number of active agents in networks wherein agents are allowed to join or leave, while maintaining hidden their identity within the network. The problem is solved by making the agents generate random numbers and infer the number of the agents that took part in the experiment by exploiting the knowledge of the maximum generated number. The main challenge and the **innovative aspect** of our problem formulation is that due to the time-varying composition of the network, then the maximum number constitutes a time-varying signal which has to be estimated in a distributed way by the agents in the network. The current literature lacks of such a protocol and this constitutes the of our approach and thus the **theoretical contribution** is the development of two different protocols solving for the first time the dynamic max-consensus and their characterization in terms of tracking and steady state errors. Other **potential applications** of the these dynamic max-consensus protocols involve distributed synchronization (e.g., time-synchronization, target tracking) and network parameter estimation (e.g., diameter and highest degree).

In many monitoring applications the full state information of a large-scale network is neither necessary nor possible, while the estimation of some aggregated quantities of the state when only a few agents' states are accessible to a centralized observer is usually sufficient and more practicable. **Potential applications** range across several contexts, such as the monitoring of the mean traffic density, the mean proportion of infected people during the spreading of an epidemic, the mean circulation of leaders opinions over geographic areas, and so on. The thesis builds on the projection strategy, which maps the dynamics of the original system into a lower dimensional state space, and the **theoretical contribution** consists in the derivation of necessary and sufficient conditions for the existence of linear and sliding mode functional observers enabling the estimation of the average state. The **innovative aspects** of the proposed designs is that their dimension does not scale with the dimension of the system and their convergence rate can be arbitrarily tuned by trading-off the steady-state error.