Resilient Self-Organizing Networks in Multi-Agent Systems via Approximate Random k-Regular Graphs

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Outline

1. Problem statement and motivation
2. Distributed algorithm to self-organize a graph into a k-regular graphs
3. Numerical simulations
4. Conclusions and future perspectives

Outline

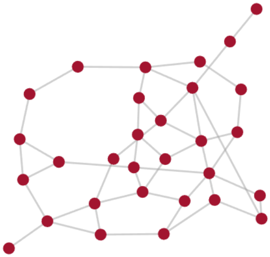
Scenarios

Peer-to-Peer Networks

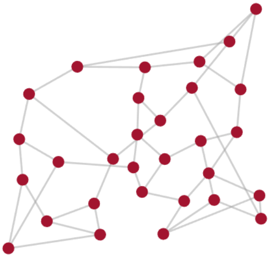
Wireless Sensor Networks

Multi-Robot Systems

k-regular networks



(a) Network with average degree 3



(b) 3-regular network: each node has exactly 3 edges

Same number of edges but different resilience to structural and functional perturbations.

Literature

k-CONNECTED GRAPHS

- Jamakovic and Uhlig, "On the relationship between the algebraic connectivity and graph's robustness to node and link failures", in *Conf. Next Gener. Internet Netw.* (2007) pp. 96–102.
- Szczytowski, Khelil, and Suri, "DKM: Distributed k-connectivity maintenance in wireless sensor networks", in *9th WONS* (2012)

r-ROBUST GRAPHS

- Zhang, Fata,and Sundaram, S, "A notion of robustness in complex networks", in *IEEE Trans. Control Netw. Syst* (2015)
- Saulnier, Saldana, Prorok, Pappas, and Kumar, "Resilient flocking for mobile robot teams", in *IEEE Robotics and Automation letters* (2017)
- Guerrero-Bonilla, Prorok, and Kumar, "Formations for resilient robot teams", in *IEEE Rob. Aut. Lett.* (2017)

k-REGULAR GRAPHS

- Yazıcıo, Egerstedt, and Shamma, "Decentralized degree regularization for multi-agent networks", in *52nd IEEE CDC* (2013)
- Yazıcıo, Egerstedt, and Shamma, "Decentralized formation of random regular graphs for robust multi-agent networks", in *53rd IEEE CDC* (2014)
- Yazıcıo, Egerstedt, and Shamma, "Formation of robust multi-agent networks through self-organizing random regular graphs", in *Trans. on Net. Sc. and Eng.* (2015)

Problem of interest and contribution

Problem

Design a local update rule to iteratively self-organize any connected network into a (*approximate*) *random* k-regular graph under the following assumption:

- each node can add edges to 2-hop neighbors.

Contributions

The proposed procedure has the following advantages with respect to the current literature:

- **Arbitrary order k**: the desired order k of regularity does not depend on the initial graph configuration;
- **Faster convergence**: the desired graph configuration is achieved in fewer steps;
- **Resilience to open networks**: the joining or leaving event the agents do not require the re-initialization of the procedure.

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Problem set-up

undirected network → $\mathcal{G} = (\mathcal{V}, \mathcal{E})$

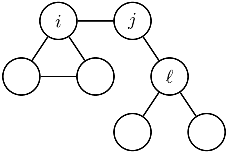
set of agents → $\mathcal{V} = \{1, \dots, n\}$

set of connections → $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$

1-hop neighbors of agent i → $\mathcal{N}_i = \{j \mid (i,j) \in \mathcal{E}\}$

2-hop neighbors of agent i → $\mathcal{N}_i^2 = \{\ell \mid \ell \in \mathcal{N}_j,\ j \in \mathcal{N}_i\}$

degree of agent i → $d_i = |\mathcal{N}_i|$



Definition: Random k-regular graphs

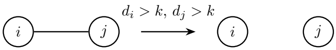
An undirected and connected graph is:

- k-regular if each node has exactly k neighbors, i.e., $d_i = k$ for all $i \in \mathcal{V}$;
- approximate k-regular if only one node has a number of neighbors different from k;
- random k-regular if it is selected uniformly at random from all k-regular graphs with the same number of nodes.

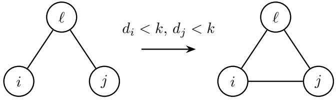
Available actions to the agents

Assumptions

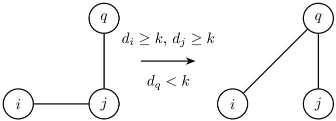
- The initial network is connected
- Each agent can add/delete edges with its 1-hop and 2-hop neighbors



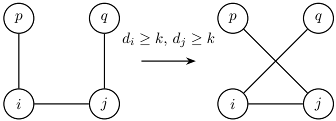
Action (R): Remove an edge



Action (A): Add an edge



Action (M1): Move an edge



Action (M2): Move an edge

Distributed Formation and Maintenance of Random k-Regular Graph

Sketch of Algorithm 1:

(Input): Order $k \in \mathbb{N}_+$ of regularity
(Execution): at each step $t = 1, 2, 3, \cdots$

- $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is the current graph
- $\mathcal{V}_a \subseteq \mathcal{V}$ is the set of randomly activated nodes
- **each node** $i \in \mathcal{V}_a$ **does**
  - **if** *action (A) is feasible* **then**
    - Execute action (A) with a random $j \in \mathcal{N}_i$
  - **else if** *action (R) is feasible* **then**
    - Execute it with a random $j \in \mathcal{N}_i^2$
  - **if** *action (M1) is feasible* **then**
    - Execute it with random $j \in \mathcal{N}_i$, $q \in \mathcal{N}_j \setminus \{i\}$
  - **else if** *action (M2) is feasible* **then**
    - Execute it with random $j \in \mathcal{N}_i$, $q \in \mathcal{N}_j \setminus \{i\}$, $p \in \mathcal{N}_i \setminus \{j\}$

Main results

Consider a network with a fixed number of agents that implements Algorithm 1 and the metric

$$f(\mathcal{G}, k) = \| \max_{i \in \mathcal{V}} \{d_i\} - k \| + \| k - \min_{i \in \mathcal{V}} \{d_i\} \|.$$

For any order $k \geq 2$, if the graph is initially connected graph and remains connected thereafter, then at each step it holds that:

i) the metric $f(\mathcal{G}, k)$ is non-increasing;

ii) if $\mathcal{G}$ is not an (approximate) k-regular graph, there is a set of agents that makes the metric $f(\mathcal{G}, k)$ decrease;

iii) if an agent joins or leaves the network, Algorithm 1 does not need to be re-initializated.

Conjectures

1. if $\mathcal{G}$ is an (approximate) k-regular graph, then the iterative execution of Algorithm 1 makes the graph a random (approximate) k-regular graph;
2. the probability of getting the graph disconnected goes to zero as the number of nodes n and the degree k goes to infinity.

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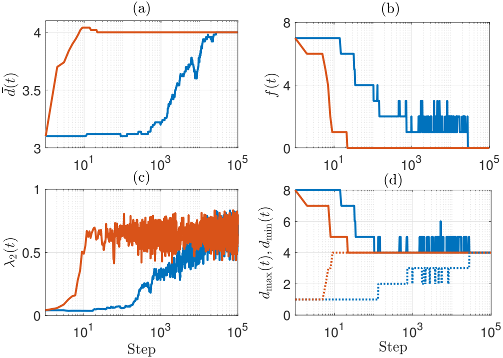
Static networks and comparison with the state-of-art

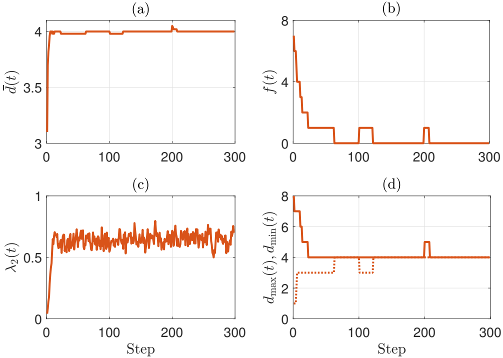
$\overline{d}$ → average degree
f → metric of regularity
λ_2 → algebraic connectivity
$d_{\min}$, $d_{\max}$ → min/max degree



Blue curve: A. Y. Yazıcıo glu, M. Egerstedt, and J. S. Shamma, "Formation of robust multi-agent networks through self-organizing random regular graphs", in *IEEE Transactions on Network Science and Engineering* (2015).

Open networks

$\overline{d}$ → average degree
f → metric of regularity
λ_2 → algebraic connectivity
$d_{\min}$, $d_{\max}$ → min/max degree



Outline

Conclusions and future directions

Contributions:

- A distributed protocol to self-organize any network into a regular graph with an arbitrary degree;
- Resiliency to open networks;

Future works:

- Provide a formal proof of randomness of the final time-varying network;
- Characterize the rate of convergence;
- ...

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Thank you for your attention!

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