Distributed tracking of graph parameters in anonymous networks with time-varying topology

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IEEE 60th Conference on Decision and Control (CDC), 13-17 December 2021

Outline

1. Problem statement and motivation
2. Dynamic max-consensus based approach
3. Numerical results and conclusions

Outline

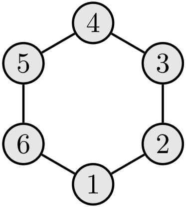
Problem statement

Problem of interest

Estimate and track the time-varying parameters in networks whose topology changes over time:

- Eccentricities of nodes;
- Diameter and radius of the network.

- Discrete time framework: $k \in \mathbb{N}$;
- Time-varying undirected graph $\mathcal{G}(k) = (V, E(k))$;
- A constant number of agents equal to $n = |V| \in \mathbb{N}$;



Parameters of interest

Let $\text{dist}_{ij}(k)$ denote the length of the shortest path between agents i and j at time k, then:

- The eccentricity $e_i(k)$ of node $i \in V$ at time k is defined as the maximal distance from i of any other node,

$$e_i(k) = \max_{j \in V} \text{dist}_{ij}(k).$$

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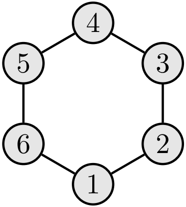

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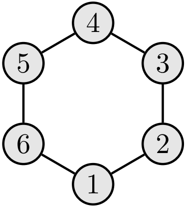

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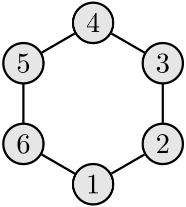

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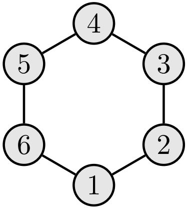

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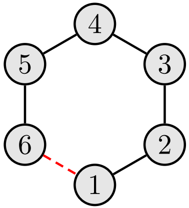

$$e_i(k^*) = \cdots, \quad \forall i \in V$$

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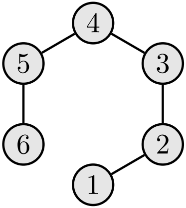

$$e_1(k^*) = e_6(k^*) = 5$$
$$e_2(k^*) = e_5(k^*) = 4$$
$$e_3(k^*) = e_4(k^*) = 3$$
$$d(k^*) = 5, \qquad r(k^*) = 3$$

Applications

Possible applications of such metrics are straightforward:

- Leader election for maximizing the spread of influence in social networks;
  [D. Kempe et al., "*Maximizing the spread of influence through a social network*" , 2003.]
- Optimal routing in large-scale networks;
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- Efficiency maintenance in wireless networks;
  [L. M. Feeney, "*Energy efficient communication in ad hoc wireless networks*" , 2004.]
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Outline

Current literature: max-consensus

Algorithms to solve the problem in static networks have been provided by resorting to the max-consensus protocol

$$x_i(k) = \max_{j \in \mathcal{N}_i \cup \{i\}} \{x_j(k-1)\}, \qquad x_i(0) = u_i,$$

where $u_i \in \mathbb{R}$ are the constant reference signals.

Key features:

- Low computational capability;
- Anonymity of the agents;
- Finite-time convergence;
- Zero steady-state error.

F. Garin, D. Varagnolo, and K. H. Johansson, "*Distributed estimation of diameter, radius and eccentricities in anonymous networks*", inIFAC Proceedings Volumes, vol. 45, Elsevier, 2012, pp. 13–18.

G. Oliva, R. Setola, and C. N. Hadjicostis, "*Distributed finite-time calculation of node eccentricities, graph radiusand graph diameter*", Systems & Control Letters, vol. 92,pp. 20–27, 2016.

Our approach: dynamic max-consensus

We provide an algorithm for dynamic networks by resorting to the dynamic max-consensus protocol

$$x_i(k) = \max_{j \in \mathcal{N}_i \cup \{i\}} \{x_j(k-1) - \alpha, u_i(k)\}, \qquad \alpha \geq 0,$$

where $u_i(k) \in \mathbb{R}$ are time-varying reference signals.

Key features:

- Low computational capability;
- Anonymity of the agents;
- Finite-time convergence;
- Bounded tracking and steady-state error;
- Robustness to re-initialization.

D. Deplano, M. Franceschelli, and A. Giua, "*Dynamic min and max consensus and size estimation of anonymous multi-agent networks*", IEEE Transaction on Automatic Control (conditionally accepted, preliminary version available on arXiv:2009.03858).

Eccentricity-Diameter-Radius (EDR) Algorithm

1. **Initialization**: Each agent $i \in V$ selects $L \in \mathbb{N}_+$ numbers with uniform distribution

$$u_{i\ell} \sim \mathcal{U}(0,1), \qquad \ell = 1, \dots, L.$$

2. **Execution**: Each agent $i \in V$ updates its states $[x, y, z]^\top \in \mathbb{R}^{3L}$ according to

$$x_{i\ell}(k) = \max_{j \in \mathcal{N}_i(k-1) \cup \{i\}} \left\{ x_{j\ell}(k-1) \right\}, \qquad x_{i\ell}(0) = u_{i\ell},$$

$$y_{i\ell}(k) = \max_{j \in \mathcal{N}_i(k-1) \cup \{i\}} \left\{ y_{j\ell}(k-1) - \alpha, u_{i\ell} \right\}, \qquad y_{i\ell}(0) = u_{i\ell},$$

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3. **Output**: Each agent $i \in V$ estimates the graph parameters as follows:

Eccenticity: $$\hat{e}_i(k) = \max_{\ell=1,\dots,L} \frac{|x_{i\ell}(k) - y_{i\ell}(k)|}{\alpha},$$

Diameter: $$\hat{d}_i(k) = \max_{\ell=1,\dots,L} \left[z_{i\ell}(k) \right],$$

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Steady-state expected errors

Assumption

There exists a minimum dwell time $\Upsilon \in \mathbb{N}_+$ between two changes of the graph $\mathcal{G}(k)$.

Theorem

Consider a network that implements the EDR Algorithm and let:

- $k_0 \in \mathbb{N}$ be a generic instant of time at which the graph changes;
- $T_c(k_0) \in \mathbb{N}$ be the time within which all max-consensus protocols converge.

If $\Upsilon \geq T_c(k_0)$ and if $\alpha < 1/d(k_0)$, then for $k \in [k_0 + T_c(k_0), k_0 + \Upsilon]$ the expected estimation errors are:

$$\mathbb{E}[e_i(k) - \hat{e}_i(k)] = \sum_{\varepsilon=1}^{e_i} \left(1 - \frac{\sum_{h=\varepsilon}^{e_i} |\mathcal{N}_i^h(k)|}{|V|}\right)^L \quad (1)$$

$$\mathbb{E}[d(k) - \hat{d}_i(k)] = \sum_{\delta=r+1}^{d} \left(1 - \frac{\sum_{h=\delta}^{d} |\mathcal{N}^h(k)|}{|V|}\right)^L \quad (2)$$

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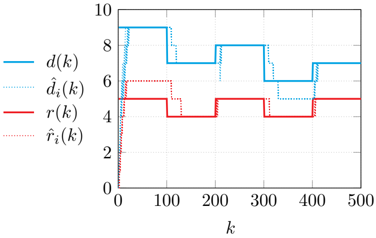
Dynamic tracking of diameter, and radius in a random network

- Number of nodes $n = 300$;
- Design parameter $\alpha = 0.1$;
- Dwell-time $\Upsilon = 100$;
- Selected numbers $L = 50$.



Remark

Every time the network changes, the dynamic max-consensus protocol allows to recover the tracking without the need of reinitialize the algorithm.

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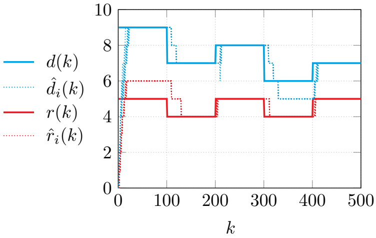

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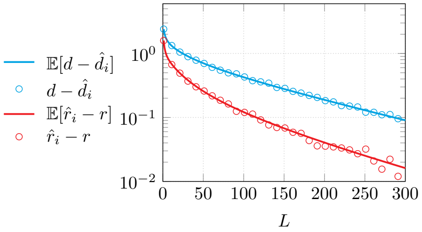

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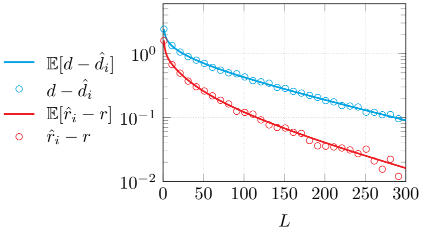

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Conclusions and future directions

Contributions:

- We have introduced a distributed algorithm to estimate and track useful parameters over time-varying networks:
- The algorithm has been proved to converge in finite-time and its expectation errors have been characterized;
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D. Deplano, M. Franceschelli, and A. Giua, "*Dynamic min and max consensus and size estimation of anonymous multi-agent networks*", IEEE Transaction on Automatic Control (conditionally accepted).

Distributed tracking of graph parameters in anonymous networks with time-varying topology

Thank you for your attention!

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