

Distributed Energy and Thermal Management of Buildings with High-Order RC Thermal Dynamics

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Abstract—Energy and thermal management of a multi-zone building equipped with photovoltaic generation, battery energy storage system, and heating, ventilation, and air conditioning system is investigated. Building thermal dynamics are modeled by a Resistance-Capacitance network accounting for shared-wall coupling between zones. A multi-objective mixed-integer linear programming (MILP) problem is formulated first to minimize electricity cost and thermal discomfort via weighted-sum scalarization, where binary variables enforce non-simultaneous grid import/export and battery charge/discharge. A formal proof of convex relaxation of the MILP into Linear Programming (LP) problem is given. Then, a convex distributed resource allocation problem solved via the Dual Consensus Alternating Direction Method of Multipliers (DC-ADMM) is studied. Simulations on a five-zone building confirm that the LP matches the MILP optimum and that DC-ADMM converges to the centralized solution, validating the convex relaxation.

Index Terms—Thermal discomfort, Convex relaxation, Distributed optimization, DC-ADMM.

I. INTRODUCTION

Global energy demand has increased substantially in recent years. Buildings account for 34% of global energy consumption, with heating, ventilation, and air conditioning (HVAC) systems representing a dominant share of used energy [1]. Therefore, an important challenge in building energy management is to reduce energy costs while maintaining thermal comfort for occupants, also in the presence of modeling uncertainties and time-varying operating conditions [2], [3], [4]. This paper proposes a multi-objective optimization to minimize total electricity cost and occupant thermal discomfort which needs building's thermal dynamics model. Resistance-Capacitance (RC) network models are widely adopted on real-time building energy optimizations [5], [6]. The accuracy of RC-network models depends on the estimation of thermal resistance and capacitance parameters. Existing approaches include physics-based estimation from building characteristics and data-driven or Kalman-filter-based identification methods [7], [8], [9], [10]. Unlike lower-order models that retain only indoor-zone temperatures, the RC model adopted in this paper includes the temperatures of shared and outer walls as

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additional state variables. This allows the scheduler to account for the thermal storage capacity of the building envelope and provides a convenient representation of the thermal coupling between adjacent zones. Although higher-order models improve scheduling accuracy, they increase computational complexity [11]. To mitigate this, several works have proposed multi-agent architectures considering each thermal zone is modeled as an agent that cooperates with others to satisfy objectives which can be solved through distributed optimization method [11], [12], [13]. Building energy management and HVAC control problems are commonly cast as mixed-integer linear programming (MILP) problems to model the binary decisions of actuators [14], [15], [16]. However, MILP problems are not directly compatible with standard distributed optimization methods [17], [18]. A standard approach to address this limitation is convex relaxation, which results in Linear Programming (LP) one [19], [20]. Here, we prove by contradiction that the convex relaxation of the MILP to the LP problem is exact; consequently, the binary constraints do not affect the optimal solution and can be skipped. Among distributed optimization methods, ADMM has been widely employed due to its decomposability and convergence guarantees for convex problems [21], [22]. However, standard ADMM exhibits slow convergence when applied to problems with coupled constraints [23]. Therefore, a variant known as Dual Consensus ADMM (DC-ADMM) has been proposed, which achieves faster convergence and exact solutions [24].

The **main contributions** of this manuscript are threefold:

- 1) Introduces a high-order RC thermal model that augments standard zone-temperature dynamics with shared and outer wall temperatures, improving the representation of heat transfer and thermal inertia in multi-zone buildings;
- 2) Establishes the exactness of the binary relaxation, showing that the MILP can be reduced to an LP without loss of optimality and without constraint violation, under mild assumptions on electricity prices, battery efficiency, and grid sizing (Assumption 1).
- 3) Derives an equivalent, fully distributed optimization framework for coordinated HVAC-BESS-grid scheduling, and demonstrates numerically that the distributed solution matches the centralized optimum.

Structure of the paper. Section II formulates the multi-objective MILP optimization. Section III establishes the binary relaxation proofs. Section IV develops the distributed optimization framework. Section V presents simulation results, and Section VI presents some concluding remarks.

II. PROBLEM FORMULATION

We consider a building composed of n thermal zones, indexed by the set $\mathcal{V} := \{1, \dots, n\}$. The zones are thermally coupled through shared walls according to an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with line topology, namely $\mathcal{E} = \{\{i, i+1\} : i = 1, \dots, n-1\}$. Accordingly, the neighbor set of zone i is defined as $\mathcal{N}_i := \{j \in \mathcal{V} : \{i, j\} \in \mathcal{E}\}$, so that $\mathcal{N}_1 = \{2\}$, $\mathcal{N}_n = \{n-1\}$, and $\mathcal{N}_i = \{i-1, i+1\}$ for all $i \in \{2, \dots, n-1\}$. The building is equipped with a common PV generation system, a centralized BESS, and one HVAC unit per zone. At each sampling instant, the energy management system determines the HVAC electrical power of each zone, the battery charging/discharging powers, and the grid power exchange, while satisfying the thermal dynamics, comfort constraints, and network power balance. Let $\mathcal{T} := \{0, \dots, N-1\}$ denote the prediction horizon, with sampling time Δ [h]. The time-invariant parameters used in the formulation are summarized in Table I. The known time-varying exogenous inputs are: P_t^{PV} and $P_{\max}^{PV}$, the aggregate PV power generation and its maximum such that $P_t^{PV} \leq P_{\max}^{PV}$; $P_{i,t}^{load}$, the non-HVAC electrical demand of zone i ; $\dot{Q}_{i,t}^{int}$, the internal heat gain in zone i ; Q_t^{sun} , the solar heat gain; T_t^{out} , the outdoor temperature; c_t^b and c_t^s , the electricity purchase and selling prices; $\alpha_t^c \in \{0, 1\}$, a known occupancy-dependent weight activating the comfort penalty during working hours when people are inside. The optimization variables are described in Table II and, for the sake of clarity, included in the following set:

$$x := \left\{ P_{i,t}^h, P_{i,t}^c, P_t^b, P_t^s, P_t^{bc}, P_t^{bd}, \delta_t, \beta_t, D_{i,t}, e_{t+1} \mid T_{i,t+1}^{in}, T_{i,t+1}^w, T_{ij,t+1}^{wc} \mid i \in \mathcal{V}, (i,j) \in \mathcal{E}, t \in \mathcal{T} \right\}.$$

A. Objective

The control objective is to minimize, over the prediction horizon $\mathcal{T}$, a weighted combination of the economic objective $J^e(x)$ and the thermal discomfort $J^\ell(x)$. Let $w_c > 0$ be the free weight parameter, we consider the objective

$$J(x) = w_c J^e(x) + J^\ell(x), \quad (1)$$

where, the net cost of electricity exchange with the grid is

$$J^e(x) = \sum_{t=0}^{N-1} \Delta (c_t^b P_t^b - c_t^s P_t^s), \quad (2)$$

where we make the assumption $c_t^b > c_t^s > 0$ for all $t \in \mathcal{T}$.

$J^\ell(x)$ is defined in terms of the deviation of the indoor temperature from the reference setpoint in each zone. This deviation can be naturally measured by the absolute value $|T_i^{ref} - T_{i,t}^{in}|$, which is convex but non-differentiable. In order to obtain a linear objective, we replace this term with an auxiliary variable $D_{i,t} \geq 0$, interpreted as the temperature-tracking error in zone i at time t . Thus, the discomfort term is the ℓ_1 -type aggregate temperature-tracking error

$$J^\ell(x) = \sum_{t=0}^{N-1} \sum_{i=1}^n \alpha_t^c D_{i,t}, \quad (3)$$

with the constraints

$$D_{i,t} \geq T_i^{ref} - T_{i,t}^{in}, \quad D_{i,t} \geq T_{i,t}^{in} - T_i^{ref}, \quad (4)$$

TABLE I

TIME-INVARIANT MODEL AND OPTIMIZATION PARAMETERS.

Parameter	Unit/domain	Description
Δ	h	Sampling time
N	$\mathbb{N}$	Prediction horizon length
γ^c / γ^h	$(0, \infty)$	HVAC cooling/heating conversion factor
$C_{i,t}^{in}$	kWh/ $^\circ$ C	Zone i thermal capacitance
$C_{ij}^{wc} = C_{ji}^{wc}$	kWh/ $^\circ$ C	Shared-wall capacitance
$C_{i,t}^w$	kWh/ $^\circ$ C	Outer-wall capacitance of zone i
$R_{i,t}^{in}$	$^\circ$ C/kW	Zone-wall thermal resistance
$R_{ij}^{wc} = R_{ji}^{wc}$	$^\circ$ C/kW	Shared-wall thermal resistance
$R_{i,t}^w$	$^\circ$ C/kW	Outer-wall thermal resistance
E_b	kWh	Battery energy capacity
η_c	$(0, 1)$	Charging efficiency
$e_{\min}/e_{\max}$	$[0, 1]$	SoC bounds
$P_{\max}^h / P_{\max}^c$	kW	Maximum HVAC heating/cooling power
$P_{\max}^{bc}, P_{\max}^{bd}$	kW	Battery power limits
$P_{\max}^{net}, P_{\max}^{PV}$	kW	Grid and PV exchange limits
$T_{i,t}^{in}, T_{i,t}^{out}$	$^\circ$ C	Indoor temperature bounds
T_i^{ref}	$^\circ$ C	Temperature reference for zone i
w_c	$\mathbb{R}_{>0}$	Cost-discomfort weight

TABLE II

DECISION VARIABLES OF THE CENTRALIZED OPTIMIZATION PROBLEM.

Variable	Unit/domain	Description
$P_{i,t}^h / P_{i,t}^c$	kW	HVAC electrical power of zone i
P_t^b / P_t^s	kW	Grid imported/exported power
P_t^{bc} / P_t^{bd}	kW	Battery charging/discharging power
e_t	$[0, 1]$	Battery state of charge
$D_{i,t}$	$^\circ$ C	Comfort-tracking auxiliary variable
δ_t	$\{0, 1\}$	Battery charge/discharge selector
β_t	$\{0, 1\}$	Grid import/export selector
$T_{i,t}^{in}$	$^\circ$ C	Indoor temperature of zone i
$T_{i,t}^w$	$^\circ$ C	Outer-wall temperature of zone i
$T_{ij,t}^{wc}$	$^\circ$ C	Shared-wall temperature between zones i, j

which imply that $D_{i,t} \geq |T_i^{ref} - T_{i,t}^{in}|$. Since $D_{i,t}$ is penalized in the objective function, its optimal value is driven to the smallest value greater than or equal to $|T_i^{ref} - T_{i,t}^{in}|$, i.e., at optimality the equality $D_{i,t} = |T_i^{ref} - T_{i,t}^{in}|$ holds.

B. Constraints

1) *Building thermal dynamics*: The thermal behavior of the building is modeled by a RC network as shown in Figure 1. The HVAC thermal power delivered to zone i is related to the corresponding electrical power by $\dot{Q}_{i,t}^{ac} = \gamma^h P_{i,t}^h - \gamma^c P_{i,t}^c$, where the constant coefficients $\gamma^h, \gamma^c > 0$ represent the cooling/heating conversion factors of the HVAC system, $P_{i,t}^h$ and $P_{i,t}^c$ are the electrical powers for heating and cooling, respectively, satisfying the linear constraints

$$\begin{aligned} 0 &\leq P_{i,t}^h \leq P_{\max}^h \max\{0, \text{sign}(T_i^{ref} - T_t^{out})\}, \\ 0 &\leq P_{i,t}^c \leq P_{\max}^c \max\{0, \text{sign}(T_t^{out} - T_i^{ref})\}. \end{aligned} \quad (5)$$

enforcing that $P_{i,t}^c$ and $P_{i,t}^h$ cannot simultaneously be non zero.

The temperature of each shared wall $(i, j) \in \mathcal{E}$ evolves as

$$T_{ij,t+1}^{wc} = T_{ij,t}^{wc} + \frac{\Delta}{C_{ij}^{wc} R_{ij}^{wc}} (T_{i,t}^{in} + T_{j,t}^{in} - 2T_{ij,t}^{wc}). \quad (6)$$

By construction, it must hold that

$$T_{ij,t}^{wc} = T_{ji,t}^{wc}. \quad (7)$$

The indoor temperature of each zone $i \in \mathcal{V}$ evolves as

$$T_{i,t+1}^{in} = T_{i,t}^{in} + \frac{\Delta}{C_i^{in}} \left(\frac{T_{i,t}^w - T_{i,t}^{in}}{R_i^w + R_{i,t}^{in}} + \sum_{j \in \mathcal{N}_i} \frac{T_{ij,t}^{wc} - T_{i,t}^{in}}{R_{ij}^{wc}} + Q_t^* \right), \quad (8)$$

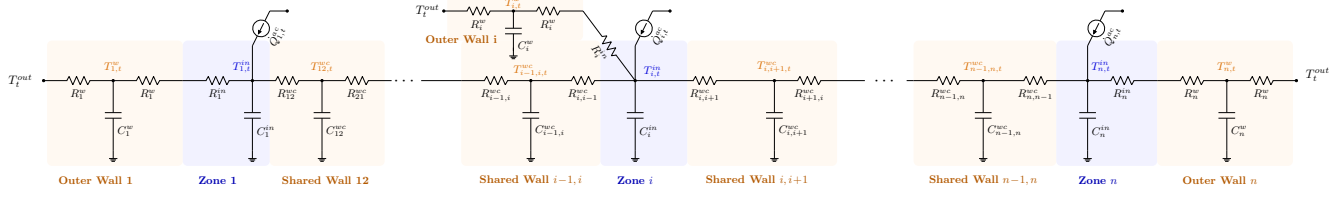


Fig. 1. RC network of thermal behavior of a building with $n \in \mathbb{N}$ consecutive zones.

where $Q_t^* = \dot{Q}_{i,t}^{ac} + \dot{Q}_{i,t}^{int} + Q_t^{sun}$, whereas the outer-wall temperature of zone $i \in \mathcal{V}$ evolves as

$$T_{i,t+1}^w = T_{i,t}^w + \frac{\Delta}{C_i^w} \left(\frac{T_{i,t}^{in} - T_{i,t}^w}{R_i^{in} + R_i^w} + \frac{T_t^{out} - T_{i,t}^w}{R_i^w} \right). \quad (9)$$

The above state equations are enforced together with the known initial conditions $T_{i,0}^{in}$, $T_{i,0}^w$, and $T_{ij,0}^{wc}$.

2) **BESS and power-balance constraints:** The BESS state of charge evolves according to

$$e_{t+1} = e_t + \frac{\Delta}{E_b} (\eta_c P_t^{bc} - P_t^{bd}). \quad (10)$$

The SoC is constrained by

$$e_{\min} \leq e_{t+1} \leq e_{\max}, \quad (11)$$

and the terminal condition

$$0.95 e_0 \leq e_N \leq 1.05 e_0, \quad (12)$$

which approximately enforces energy neutrality over the horizon. To prevent simultaneous battery charging and discharging, we introduce the binary variable $\delta_t \in \{0, 1\}$ and impose

$$0 \leq P_t^{bc} \leq P_{\max}^{bc} \delta_t, \quad 0 \leq P_t^{bd} \leq P_{\max}^{bd} (1 - \delta_t). \quad (13)$$

Similarly, to prevent simultaneous grid import and export, we introduce the binary variable $\beta_t \in \{0, 1\}$ and enforce

$$0 \leq P_t^b \leq P_{\max}^{net} \beta_t, \quad 0 \leq P_t^s \leq P_{\max}^{net} (1 - \beta_t). \quad (14)$$

The admissible indoor temperature range is

$$T_{\min}^{in} \leq T_{i,t+1}^{in} \leq T_{\max}^{in}. \quad (15)$$

We adopt the convention that all power variables are non-negative. At each time step, P_t^b , P_t^{PV} , and P_t^{bd} represent power supplied to the building by grid import, PV generation, and battery discharge, respectively. Conversely, $P_{i,t}^h + P_{i,t}^c$, $P_{i,t}^{load}$, P_t^{bc} , and P_t^s appear on the demand/export side of the balance, corresponding to HVAC consumption, non-HVAC load, battery charging, and grid export, respectively. Therefore, the electrical power balance is written as

$$P_t^b + P_t^{PV} + P_t^{bd} = \sum_{i=1}^n (P_{i,t}^h + P_{i,t}^c + P_{i,t}^{load}) + P_t^{bc} + P_t^s. \quad (16)$$

C. MILP formulation

The resulting MILP optimization problem given by

$$\begin{aligned} \min_x \quad & J(x) \text{ as in eq. (1)} \\ \text{s.t.} \quad & \text{const. in eqs. (4)-(16), } \forall i \in \mathcal{V}, \forall (i,j) \in \mathcal{E}, \forall t \in \mathcal{T}. \end{aligned} \quad (17)$$

III. EXACT BINARY RELAXATION

This section studies when the binary variables in (17) are redundant at optimality. Consider the LP relaxation obtained by removing the binary variables δ_t and β_t and replacing (13) and (14), respectively, with the box constraints

$$0 \leq P_t^{bc} \leq P_{\max}^{bc}, \quad 0 \leq P_t^{bd} \leq P_{\max}^{bd}, \quad (18)$$

$$0 \leq P_t^b \leq P_{\max}^{net}, \quad 0 \leq P_t^s \leq P_{\max}^{net}. \quad (19)$$

In particular, the next theorem shows that the binary variables enforcing non-simultaneous grid import/export and non-simultaneous battery charge/discharge are redundant at optimality under the following mild assumption.

Assumption 1. *The MILP (17) is feasible and the grid exchange capacity can absorb peak PV generation together with full battery discharge, namely $P_{\max}^{PV} + P_{\max}^{bd} \leq P_{\max}^{net}$.*

Remark 1. *Assumption 1 is a mild grid-sizing condition requiring the grid connection to handle peak PV output plus maximum battery discharge simultaneously, which is standard practice in building microgrid design.*

Therefore, under Assumption 1, the MILP in (17) admits an exact LP relaxation, which is the formulation used in the distributed optimization framework of Section IV.

Theorem 1. *Consider the MILP (17) under Assumption 1. If $\eta_c \in (0, 1)$ and $c_t^b > c_t^s > 0$ for all $t \in \mathcal{T}$, then any optimal solution of the LP relaxation, where constraints (13)-(14) are replaced by the relaxed constraints (18)-(19), does not allow simultaneous (i) grid import/export, i.e., $P_t^b P_t^s = 0$, and (ii) battery charge/discharge $P_t^{bc} P_t^{bd} = 0$, at every time $t \in \mathcal{T}$.*

Proof: The variables P_t^b , P_t^s , P_t^{bc} , and P_t^{bd} do not appear in J^ℓ . Therefore, in the following proofs, only J^e needs to be examined. We first carry out the proof of statement (i). Fix any $t \in \mathcal{T}$ and suppose, by contradiction, that optimal solutions satisfy $P_t^b > 0$ and $P_t^s > 0$. Define $\varepsilon_t := \min\{P_t^b, P_t^s\} > 0$ and construct the alternative variables

$$\hat{P}_t^b := P_t^b - \varepsilon_t, \quad \hat{P}_t^s := P_t^s - \varepsilon_t. \quad (20)$$

All the other decision variables are left unchanged. The pair $(\hat{P}_t^b, \hat{P}_t^s)$ is feasible, because $0 \leq \hat{P}_t^b \leq P_t^b \leq P_{\max}^{net}$ and $0 \leq \hat{P}_t^s \leq P_t^s \leq P_{\max}^{net}$. Moreover, relation $\hat{P}_t^b - \hat{P}_t^s = P_t^b - P_t^s$ holds, hence the power-balance constraint (16) is preserved. Let $J_t^e := \Delta (c_t^b P_t^b - c_t^s P_t^s)$ denote the contribution of time step t to J^e . Under the modified variables in (20), this term becomes $\hat{J}_t^e = J_t^e - \Delta \varepsilon_t (c_t^b - c_t^s)$. Since $\varepsilon_t > 0$ and $c_t^b > c_t^s$, it follows that $\hat{J}_t^e < J_t^e$. Therefore, the modified point is feasible and yields a strictly smaller objective value, which

contradicts optimality. Hence $P_t^b P_t^s = 0$ for all $t \in \mathcal{T}$, completing the proof of statement (i).

We now carry out the proof of statement (ii). Suppose, by contradiction, that at some $t \in \mathcal{T}$ optimal solutions satisfy $P_t^{bc} > 0$ and $P_t^{bd} > 0$. Construct an alternative solution that preserves the net SoC increment, $\eta_c \tilde{P}_t^{bc} - \tilde{P}_t^{bd} = \eta_c P_t^{bc} - P_t^{bd}$, thereby leaving the SoC trajectory unchanged and preserving constraints (11)–(12). Two cases must be considered:

Case 1 (net charging): $\eta_c P_t^{bc} - P_t^{bd} \geq 0$. Set

$$\tilde{P}_t^{bd} := 0, \quad \tilde{P}_t^{bc} := P_t^{bc} - \frac{1}{\eta_c} P_t^{bd} \geq 0. \quad (21)$$

Case 2 (net discharging): $\eta_c P_t^{bc} - P_t^{bd} < 0$. Set

$$\tilde{P}_t^{bc} := 0, \quad \tilde{P}_t^{bd} := P_t^{bd} - \eta_c P_t^{bc} > 0. \quad (22)$$

Power balance: In *Case 1*, the modified battery powers preserve the same SoC increment but change the battery term in the power balance from $P_t^{bc} - P_t^{bd}$ to $\tilde{P}_t^{bc} - \tilde{P}_t^{bd} = P_t^{bc} - P_t^{bd} - P_t^{bd}/\eta_c$. Hence, the required net grid import decreases by $\sigma_t := (1/\eta_c - 1)P_t^{bd} > 0$. Accordingly, one way to adjust the grid powers is the following

$$\tilde{P}_t^b := \max\{0, P_t^b - \sigma_t\}, \quad \tilde{P}_t^s := P_t^s + \sigma_t - (P_t^b - \tilde{P}_t^b),$$

Then, by construction, replacing P_t^{bc} , P_t^{bd} , P_t^s , P_t^b with the modified powers $\tilde{P}_t^{bc}$, $\tilde{P}_t^{bd}$, $\tilde{P}_t^s$, $\tilde{P}_t^b$ preserves the power balance (16). We next prove that the modified grid powers are feasible as they satisfy the constraints (19). First, $0 \leq \tilde{P}_t^b \leq P_t^b \leq P_{\max}^{net}$ and $0 \leq \tilde{P}_t^s$ hold by construction. Second, we prove $\tilde{P}_t^s \leq P_{\max}^{net}$ by exploiting statement (i). If $P_t^b \geq \sigma_t > 0$, then $\tilde{P}_t^b > 0$, and statement (i) gives $P_t^s = 0$. Hence, $\tilde{P}_t^s = P_t^s = 0$, so the upper bound is satisfied trivially. Otherwise, $\tilde{P}_t^b = 0$ and the power balance gives $\tilde{P}_t^s = P_t^{PV} - \sum_{i=1}^n (P_{i,t}^c + P_{i,t}^h + P_{i,t}^{load}) - \tilde{P}_t^{bc} \leq P_t^{PV} \leq P_{\max}^{net}$, where the last inequality follows from Assumption 1. Finally, using (21), the per-step electricity cost satisfies

$$\tilde{J}_t^e - J_t^e \leq -\Delta P_t^{bd} \left(\frac{1}{\eta_c} - 1 \right) c_t^s < 0,$$

since $\eta_c \in (0, 1)$, $P_t^{bd} > 0$, and $c_t^s > 0$. Hence, the modified solution is feasible and attains a strictly smaller value of J^e , contradicting optimality.

A similar argument applies to *Case 2*. In that case, the surplus is $\sigma_t := (1 - \eta_c)P_t^{bc} > 0$, and the modified export satisfies $\tilde{P}_t^s \leq P_t^{PV} + P_{\max}^{bd} \leq P_{\max}^{net}$ by Assumption 1. ■

IV. DISTRIBUTED OPTIMIZATION FORMULATION

The exact LP relaxation established in Theorem 1 allows the building energy-management problem to be reformulated as a convex distributed optimization problem. The term distributed refers to the fact that the zones do not optimize independently. Instead, they cooperate through an iterative process, where each iteration consists of a computation step and a communication step with neighboring zones, eventually steering each zone toward the global optimal solution. We associate one agent with each thermal zone, and we let the communication graph coincide with the thermal coupling graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \dots, n\}$ and $\mathcal{N}_i := \{j \in \mathcal{V} : \{i, j\} \in \mathcal{E}\}$ denotes the neighbor set of zone i .

The only couplings that prevent a fully separable formulation are: (i) the shared-wall temperatures between adjacent zones, and (ii) the common electrical resources, namely grid exchange and BESS operation. To overcome this issue, we next reformulate the optimization problem by introducing local variables and coupling constraints for the shared and common resources, without assigning fixed shares to the zones in advance, thus preserving the optimal solution of the original problem while making it amenable to a distributed solution.

For each zone $i \in \mathcal{V}$, we introduce local nonnegative allocations $P_{i,t}^b$, $P_{i,t}^s$, $P_{i,t}^{bc}$, and $P_{i,t}^{bd}$, such that

$$\begin{aligned} P_t^b &= \sum_{i \in \mathcal{V}} P_{i,t}^b, \quad P_t^s = \sum_{i \in \mathcal{V}} P_{i,t}^s, \\ P_t^{bc} &= \sum_{i \in \mathcal{V}} P_{i,t}^{bc}, \quad P_t^{bd} = \sum_{i \in \mathcal{V}} P_{i,t}^{bd}, \end{aligned} \quad (23)$$

with

$$P_{i,t}^{bc} \geq 0, \quad P_{i,t}^{bd} \geq 0, \quad P_{i,t}^b \geq 0, \quad P_{i,t}^s \geq 0. \quad (24)$$

The aggregate charge/discharge and grid-exchange limits are

$$\begin{aligned} \sum_{i \in \mathcal{V}} P_{i,t}^{bc} &\leq P_{\max}^{bc}, & \sum_{i \in \mathcal{V}} P_{i,t}^{bd} &\leq P_{\max}^{bd}, \\ \sum_{i \in \mathcal{V}} P_{i,t}^b &\leq P_{\max}^{net}, & \sum_{i \in \mathcal{V}} P_{i,t}^s &\leq P_{\max}^{net}, \end{aligned} \quad (25)$$

while the electrical power balance becomes

$$P_t^{PV} = \sum_{i \in \mathcal{V}} (P_{i,t}^c + P_{i,t}^h + P_{i,t}^{load} + P_{i,t}^{bc} + P_{i,t}^s - P_{i,t}^b - P_{i,t}^{bd}). \quad (26)$$

To avoid introducing a common primal battery state in the peer-to-peer formulation, define the local net battery contribution $u_{i,t} := \eta_c P_{i,t}^{bc} - P_{i,t}^{bd}$. Then the BESS state constraints can be written equivalently as cumulative affine constraints

$$e_{\min} - e_0 \leq \frac{\Delta}{E_b} \sum_{i \in \mathcal{V}} \sum_{\tau=0}^{t-1} u_{i,\tau} \leq e_{\max} - e_0, \quad (27)$$

and the terminal condition becomes

$$-0.05 e_0 \leq \frac{\Delta}{E_b} \sum_{i \in \mathcal{V}} \sum_{\tau=0}^{N-1} u_{i,\tau} \leq 0.05 e_0. \quad (28)$$

Likewise, for every shared wall $(i, j) \in \mathcal{E}$, zones i and j maintain local copies $T_{i,j,t}^{wc}$ and $T_{j,i,t}^{wc}$ of the same wall temperature, which must thus satisfy (7).

All purely local thermal constraints remain unchanged. In particular, for each zone i , the indoor-temperature dynamics, outer-wall dynamics, comfort linearization, and temperature bounds are exactly the zone-wise instances of (4), (5), (8), (9), and (15). Similarly, for each edge $(i, j) \in \mathcal{E}$, the shared-wall dynamics are the corresponding edge-wise instances of (6).

The centralized objective separates additively across zones

$$J(\mathcal{X}) = \sum_{i \in \mathcal{V}} J_i(\mathcal{X}_i), \quad J_i(\mathcal{X}_i) = w_c J_i^e(\mathcal{X}_i) + J_i^\ell(\mathcal{X}_i),$$

with $J_i^e(\mathcal{X}_i) = \sum_{t=0}^{N-1} \Delta (c_t^b P_{i,t}^b - c_t^s P_{i,t}^s)$ and

$$J_i^\ell(\mathcal{X}_i) = \sum_{t=0}^{N-1} \alpha_t^c D_{i,t}.$$

Let x_i denote the local decision vector for zone i

$$x_i := \left[P_{i,t}^h, P_{i,t}^c, P_{i,t}^b, P_{i,t}^s, P_{i,t}^{bc}, P_{i,t}^{bd}, D_{i,t}, T_{i,t+1}^{in}, T_{i,t+1}^w, T_{i,t+1}^{wc} \mid \forall j \in \mathcal{N}_i, \forall t \in \mathcal{T} \right]^\top.$$

The distributed problem can then be written as follows

$$\begin{aligned} \min_{x_i} \quad & \sum_{i \in \mathcal{V}} J_i(x_i) \\ \text{s.t.} \quad & \text{loc. const. (4), (5), (8), (9), (15), (24)} \\ & \text{coupled const. (6), (7), (25), (26), (27), (28)} \\ & \forall i \in \mathcal{V}, \forall (i, j) \in \mathcal{E}, \forall t \in \mathcal{T}. \end{aligned} \quad (29)$$

Then, the problem (29) can be written in compact form as

$$\begin{aligned} \min_{x_i \in \mathcal{X}_i} \quad & \sum_{i \in \mathcal{V}} J_i(x_i) \\ \text{s.t.} \quad & \sum_{i \in \mathcal{V}} A_i x_i - b_i = 0, \quad \sum_{i \in \mathcal{V}} C_i x_i - d_i \leq 0, \end{aligned} \quad (30)$$

where $\mathcal{X}_i$ are local constraint sets satisfying eqs. (4), (5), (8), (9), (15), (24) and where matrices A_i, C_i and vectors b_i, d_i implement the equalities and inequalities coupled constraints in eqs. (6), (7), (25), (26), (27), (28). Furthermore, introducing dummy variables y_i for all $i \in \mathcal{V}$, the coupled inequality constraints can be equivalently replaced by local inequality constraints plus coupled equality constraints as follows

$$\begin{cases} C_i x_i + y_i - d_i \leq 0, & \forall i \in \mathcal{V} \\ \sum_{i \in \mathcal{V}} y_i = 0 \end{cases}$$

Therefore, letting $z_i = [x_i^\top, y_i^\top]^\top$ denote the new set of variables, then the problem can be written in the standard distributed *resource allocation* problem as follows

$$\min_{z_i \in \mathcal{Z}_i} \sum_{i \in \mathcal{V}} \tilde{J}_i(z_i) \quad \text{s.t.} \quad \sum_{i \in \mathcal{V}} (\tilde{A}_i z_i - \tilde{b}_i) = 0, \quad (31)$$

where $\mathcal{Z}_i$ is the intersection of $\mathcal{X}_i$ and $C_i x_i + y_i - d_i \leq 0$, while $\tilde{A}_i, \tilde{b}_i$ accounts for both A_i, b_i and $\sum_{i \in \mathcal{V}} y_i = 0$. The class of problems as in eq. (31) is decomposed into n subproblems with coupled equality constraints, and thus it can be solved, for instance, by DC-ADMM proposed in [18]; specifically, we refer to [18, Section IV.A] for details on how the algorithm is implemented, which variables need to be exchanged, and which are the design parameters.

V. NUMERICAL RESULTS

This section aims to assess the proposed framework on a common benchmark and to compare, under identical conditions, the original centralized MILP, its exact centralized LP relaxation, and the distributed implementation solved by means of the DC-ADMM protocol proposed in [18]. To this end, we consider a building with $n = 5$ consecutive zones and $\mathcal{E} = \{\{1, 2\}, \{2, 3\}, \{3, 4\}, \{4, 5\}\}$. The optimization horizon is day-ahead with $N = 24$ steps and sampling time $\Delta = 1$ h.

The multi-objective scalarization weight is $w_c = 8$. Comfort limits are $T_{in, \min} = 20^\circ\text{C}$ and $T_{in, \max} = 24.5^\circ\text{C}$, with reference temperature $T_i^{ref} = 22^\circ\text{C}$. Initial thermal states are $T_{i,0}^{in} = 21^\circ\text{C}$, $T_{i,0}^w = 19^\circ\text{C}$, and $T_{i,0}^{wc} = 19.5^\circ\text{C}$. The HVAC conversion factors are $\gamma^h = \gamma^c = 3.2$, and each zone has $P_{\max}^h = P_{\max}^c = 3.2$ kW maximum HVAC electrical power.

Thermal parameters are mildly heterogeneous across zones: $C_i^{in} = 3.00 + 0.12(i - 1)$ kWh/ $^\circ\text{C}$, $C_i^w = 8.00 + 0.10(i - 1)$ kWh/ $^\circ\text{C}$, $R_i^{in} = 1.35 + 0.06(i - 1)$ $^\circ\text{C}/\text{kW}$, and $R_i^w = 2.05 + 0.05(i - 1)$ $^\circ\text{C}/\text{kW}$ for $i \in \{1, \dots, 5\}$. Shared-wall parameters are $C_{ij}^{wc} = 6.0$ kWh/ $^\circ\text{C}$ and $R_{ij}^{wc} = 1.2$ $^\circ\text{C}/\text{kW}$.

Battery and grid limits are $E_b = 10$ kWh, $\eta_c = 0.95$, $e_0 = 0.55$ p.u., $e_{\min} = 0.20$, $e_{\max} = 0.90$, $P_{\max}^{bc} = 3.0$ kW, $P_{\max}^{bd} = 3.0$ kW, and $P_{\max}^{net} = 8.0$ kW.

All exogenous signals are deterministic and represent day-ahead predictions over the forward horizon $t \in \mathcal{T}$. Specifically, P_t^{PV} denotes the aggregate PV power generation and is modeled as a daytime sinusoid clipped at zero, with 4 kW peak. $P_{i,t}^{load}$ is the non-HVAC electrical demand of zone i , obtained as a zone-dependent base load plus an evening increment. $\dot{Q}_{i,t}^{int}$ is the internal heat-gain profile of zone i , composed of a constant baseline and an occupancy-dependent component. Q_t^{sun} is the solar heat-gain input, active during daylight hours with the same day/night pattern used for PV. T_t^{out} is the outdoor temperature profile, modeled as a smooth daily sinusoid around a cool-day mean. Electricity prices are time-varying and asymmetric: c_t^b is the purchase tariff (0.31 €/kWh for $8 \leq t < 22$, 0.20 otherwise), while c_t^s is the selling tariff (0.09 €/kWh for $11 \leq t < 16$, 0.08 otherwise). Finally, $\alpha_t^c \in \{0, 1\}$ is a known occupancy weight that activates the discomfort penalty during occupied hours ($\alpha_t^c = 1$ for $8 \leq t < 19$, 0 otherwise).

The trajectories for the tested formulations are reported in Figure 2. MILP and LP are very similar and overlap somewhere, confirming the exactness of the binary relaxation under the adopted assumptions. DC-ADMM achieves the same optimal profiles in distributed form where each zone solves iteratively a local subproblem and exchanges messages with its neighbors. The objective evolution of the DC-ADMM approach, together with the corresponding constraint violation, is reported in Figure 3. The horizontal line denotes the MILP optimum used as a reference. In early iterations, the objective value may temporarily be better than the reference because the coupling constraints are not yet satisfied. As the iterations proceed, the constraint violation decreases and the objective value approaches the reference optimum.

VI. CONCLUSIONS

This paper presented a distributed energy management framework for multi-zone buildings equipped with PV generation, BESS, and HVAC systems. A multi-objective optimization minimizing electricity cost and thermal discomfort was formulated, adopting an RC model that explicitly accounts for shared and outer wall temperatures. The centralized problem was initially posed as a MILP. Using convex relaxation, a LP problem was obtained. A key contribution is the formal proof by contradiction that the binary constraints are redundant at optimality. Then, a distributed resource allocation problem is solved via DC-ADMM, in which each zone operates as an agent coordinating with its neighbors. Simulation results confirmed the exactness of the convex relaxation and the convergence of the distributed implementation to the centralized optimum. Future work will extend the framework to handle forecast uncertainty through stochastic or robust optimization.

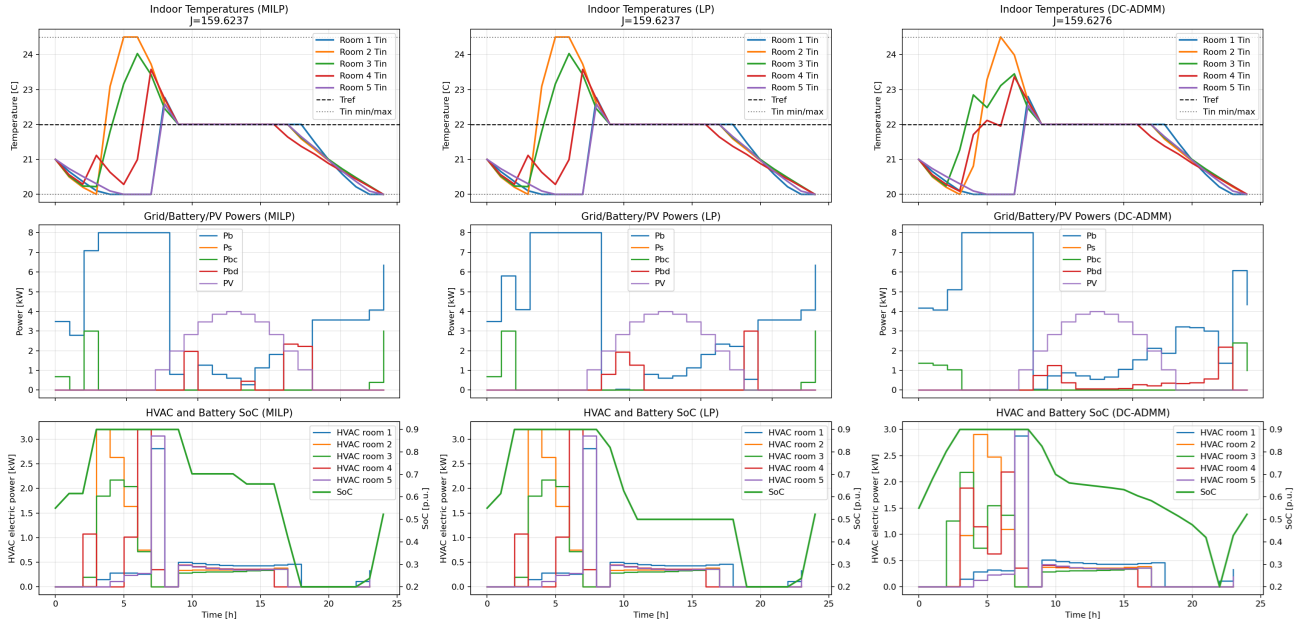


Fig. 2. Comparison of state and input trajectories for the 5-zone simulation (MILP, LP, and DC-ADMM).

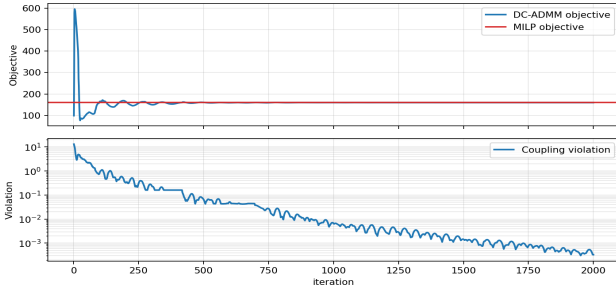


Fig. 3. Trajectories of DC-ADMM objective and constraint violation for the 5-zone case. The red horizontal line is the MILP objective reference.

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